AP® CALCULUS BC
2012 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Because $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\left.dy/dt\right|_{t=2}}{\left.dx/dt\right|_{t=2}} = 3.055$ (or 3.054)

3 : { 1 : moving to the right with reason; 1 : considers $\frac{dy/dt}{dx/dt}$; 1 : slope at $t = 2$ }

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)

2 : { 1 : integral; 1 : answer }

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

2 : { 1 : speed; 1 : acceleration }

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$
$= 0.651$ (or 0.650)

2 : { 1 : integral; 1 : answer }